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BACKSCATTERING OF RADAR WAVES FROM A TILTED, SLIGHTLY ROUGH SURFACE

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This report presents the derivation of an equation for the radar backscatter coefficient from a tilted, slightly rough surface. The basic technique employed in the solution is the small perturbation method, along with the Fourier transform. The incident wave is allowed to have an arbitrary polarization, and specific results are shown for horizontal, vertical, circular, and elliptical polarizations. One conclusion of this report is that the influence of a given slope of the tilted rough surface, upon the calculation of the radar backscatter coefficient, depends upon the polarization of the incident electromagnetic wave.

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SUMMARY

The work described in this report was done under two separate but related tasks. One task is that of generating radar image simulations of various types of terrain. To adequately simulate a radar image, appropriate radar scattering models are needed to determine the final image gray tone. The particular model developed in this report is that of a tilted, slightly rough surface. A surface such as this could simulate a road, runway, desert, or any other surface that meets the constraints of the solution. The second task is obtaining military geographic information using radar. If a calibrated radar were used to provide appropriate radar measurements of terrain and if these measurements were coupled with an applicable scattering theory, then physical and electromagnetic parameters that characterize the surface could be calculated.

Therefore, this report presents the necessary mathematics to solve the problem of electromagnetic wave scattering from a tilted, slightly rough surface. The polarization of the incident wave is allowed to be arbitrary, and specific results are shown for horizontal, vertical, circular, and elliptical polarizations.

CONTENTS

ILLUSTRATIONS

BACKSCATTERING OF RADAR WAVES FROM A TILTED, SLIGHTLY ROUGH SURFACE

INTRODUCTION

Purpose. This report develops an equation for the radar backscatter coefficient from a tilted, slightly rough surface. The basic technique employed in the solution is the small perturbation method, along with the Fourier transform.

Background. The problem of treating radar wave scattering from random rough surfaces has been studied for many years. There are many practical applications for a theory which could explain this problem. One application is the development of the basic understanding of scattering phenomena. Another application uses scattering theories, along with certain radar measurements, to determine quantitatively various physical parameters of the surface itself. For instance, if one has experimentally measured the radar backscatter coefficient at several polarizations or frequencies over a particular terrain surface, one may, by using an appropriate theory, be able to calculate surface roughness properties, moisture content, or other surface characteristics. Still another application is the area of radar image simulations. If one is interested in making a radar image simulation of a particular feature, then theoretical radar scattering models can be used, along with the radar range equation, to obtain quantitative expressions for the simulated radar image gray tones. The above-mentioned applications provide incentive for finding solutions to various radar scattering problems.

A problem in radar scattering that needs theoretical development is that of scattering from a tilted, slightly rough surface. One solution to this problem has been developed by Valenzuela.[1] However, his final result considers only the effect of a surface that is tilted in the direction orthogonal to the plane of incidence. In general, there will be two components of slope that must be considered, one component of slope in the plane of incidence and the other component in a direction orthogonal to the plane of incidence. The following derivation will consider the scattering of electromagnetic waves from a slightly rough surface that has an arbitrary tilt. The derivation will be confined to a consideration of waves in the backscatter direction only; however, the incident polarization will be taken as arbitrary. The final result of the derivation will be an equation for the radar backscatter coefficient in terms of the two slopes. The small perturbation technique will be used, and results up to first order will be considered. The derivation uses the rationalized MKS system of units. Also, results published previously will be used to obtain a final solution.[2]

[1] G. R. Valenzuela, "Scattering of Electromagnetic Waves from a Tilted, Slightly Rough Surface," *Radio Science*, Vol. 3, No. 11, November 1968.

[2] R. A. Hevenor, *Backscattering of Electromagnetic Waves from a Surface Composed of Two Types of Surface Roughness*, ETL-TR-71-4, U.S. Army Engineer Topographic Laboratories, Fort Belvoir, VA., October 1971, AD-737 675.

ANALYSIS

The geometry of the problem to be studied in this report is given in Figure 1. A right handed rectangular cartesian coordinate system (x, y, z) is set up with the xz plane coinciding with the plane of the paper. The y axis is perpendicular to the plane of the paper and pointing into it.

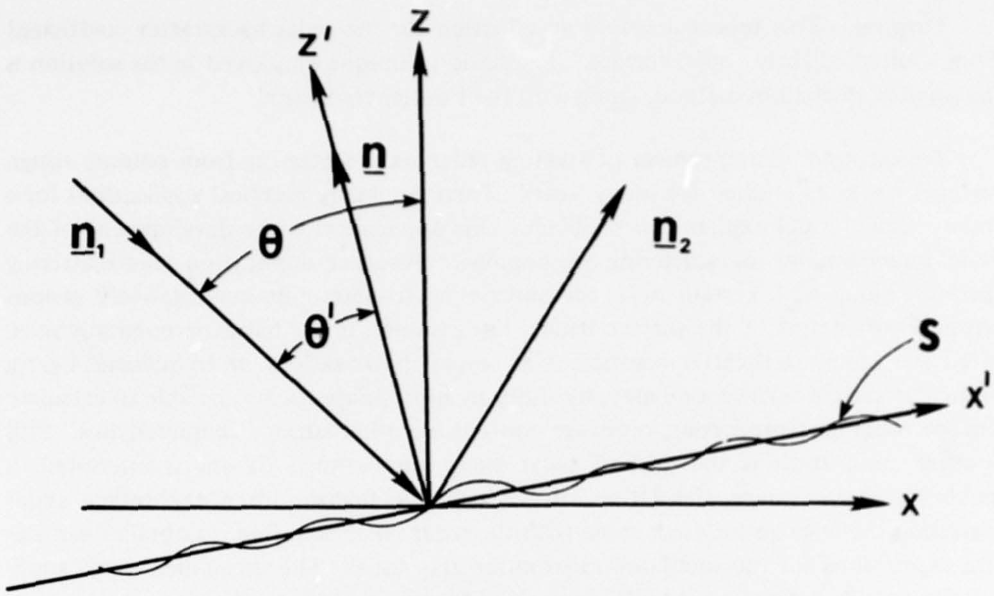

Figure 1. Scattering Geometry

A plane wave with a time harmonic of exp(jωt) is incident from free space onto a dielectric surface S. The xz plane will be taken as the plane of incidence with $\underline{n}_1$ a unit vector in the direction of propagation. An equation for $\underline{n}_1$ can be written as follows:

$$\underline{n}_1 = \underline{a}_x \sin\theta - \underline{a}_z \cos\theta \tag{1}$$

where $\underline{a}_x$ is a unit vector in the x direction.
$\underline{a}_z$ is a unit vector in the z direction.
θ is the angle of incidence measured with respect to the z axis.

The line underneath a symbol shall be used to represent a vector quantity. The surface S will be considered equal to the sum of two surfaces; one of which is a plane and the other of which is a random, slightly rough surface. If we represent the z coordinate of the surface S by ρ (x, y), then we can write the following expression for ρ (x, y):

$$\rho(x, y) = Z(x, y) + s(x, y) \quad (2)$$

where Z (x, y) represents an expression for the z coordinate of a plane that has an arbitrary tilt, and s (x, y) represents an expression for the z coordinate of a random, slightly rough surface.

The surface function s (x, y) will be considered as being generated by a wide sense stationary gaussian random process with a zero mean, a variance σ^2, and a correlation function $\phi(\tau_x, \tau_y)$. The unit vector $\underline{n}_2$ is in the direction of the receiver. For the case of a monostatic radar system, $\underline{n}_2$ is simply equal to the negative of $\underline{n}_1$. The unit vector $\underline{n}$ is an outward normal to the plane, Z (x, y) and can be written as

$$\underline{n} = \frac{-\underline{a}_x Z_x - \underline{a}_y Z_y + \underline{a}_z}{\sqrt{1 + Z_x^2 + Z_y^2}} \quad (3)$$

where $Z_x = \dfrac{\partial Z}{\partial x}$ and represents the slope of the plane in the x direction.

$Z_y = \dfrac{\partial Z}{\partial y}$ and represents the slope of the plane in the y direction.

$\underline{a}_x$, $\underline{a}_y$, and $\underline{a}_z$ are unit vectors in the x, y, and z directions, respectively.

The local angle of incidence, θ', is defined as the angle between $\underline{n}$ and $-\underline{n}_1$ and can be calculated as follows:

$$\cos\theta' = -\underline{n}_1 \cdot \underline{n} \quad (4)$$

The incident electric field ($\underline{E}_i$) can be written as

$$\underline{E}_i = \underline{e}_i \, e^{-jk_o \underline{n}_1 \cdot \hat{\underline{r}}} \quad (5)$$

where $\underline{e}_i$ is an arbitrary unit polarization vector and, in general, is complex.

j is $\sqrt{-1}$.
k_o is the free space propagation constant.
$\hat{\underline{r}}$ is a position vector equal to $x\underline{a}_x + y\underline{a}_y + z\underline{a}_z$.

The polarization vector $\underline{e}_i$ is chosen as arbitrary to consider the results for a number of different incident polarizations, such as horizontal, vertical, and circular. The time harmonic portion of the wave has been suppressed and will not be needed in the rest of the analysis.

A local rectangular coordinate system $(x'y'z')$ is set up such that the z' axis coincides with the unit normal $\underline{n}$. The y' axis (into the paper) is placed such that it is perpendicular to the local plane of incidence formed by $\underline{n}$ and $\underline{n}_1$. An equation for a unit vector in the y' direction $(\underline{a}'_y)$ would be as follows:

$$\underline{a}'_y = \underline{n} \times \underline{n}_1 / (|\underline{n} \times \underline{n}_1|) \tag{6}$$

When the definitions for $\underline{n}_1$ and $\underline{n}$ as given by equations (1) and (3) are placed in (6), the following expression for $\underline{a}'_y$ results:

$$\underline{a}'_y = [\underline{a}_x Z_y \cos\theta + \underline{a}_y (\sin\theta - Z_x \cos\theta) + \underline{a}_z Z_y \sin\theta] D_1 \tag{7}$$

where

$$D_1 = \left\{ Z^2_y + (\sin\theta - Z_x \cos\theta)^2 \right\}^{-1/2}$$

Since the direction of the z' axis coincides with $\underline{n}$, a unit vector $(\underline{a}'_z)$ in the direction of z' will simply be equal to $\underline{n}$, and a unit vector in x' can now be calculated easily.

$$\underline{a}'_x = \underline{a}'_y \times \underline{a}'_z \tag{8}$$

$$\begin{aligned} \underline{a}'_x = \Big\{ & \underline{a}_x [\sin\theta - Z_x \cos\theta + Z^2_y \sin\theta] - \underline{a}_y Z_y [\cos\theta + Z_x \sin\theta] \\ & + \underline{a}_z [Z_x (\sin\theta - Z_x \cos\theta) - Z^2_y \cos\theta] \Big\} D_0 D_1 \end{aligned} \tag{9}$$

where

$$D_0 = \left\{ 1 + Z^2_x + Z^2_y \right\}^{-1/2}$$

The problem of electromagnetic wave scattering from a slightly rough surface with no tilt has been developed for both horizontally and vertically polarized incident waves. If the electric field incident onto the tilted slightly rough surface is broken up into local horizontal and vertically polarized components, then existing results can be used to obtain a solution. The local horizontally polarized component can be found by computing the dot product of $\underline{E}_i$ with $\underline{a}'_y$.

$$\underline{E}_i \cdot \underline{a}'_y = \underline{a}'_y \cdot \underline{e}_i \, e^{-jk_o \underline{n}_1 \cdot \hat{r}} \tag{10}$$

In general, the polarization vector $\underline{e}_i$ is made up of three orthogonal components.

$$\underline{e}_i = e_{ix} \underline{a}_x + e_{iy} \underline{a}_y + e_{iz} \underline{a}_z \tag{11}$$

The local horizontally polarized component of the incident field then becomes

$$\underline{E}_i \cdot \underline{a}_y' = a_1 e^{-jk_o \underline{n}_1 \cdot \hat{\underline{r}}} \tag{12}$$

where

$$a_1 = [Z_y (e_{ix} \cos\theta + e_{iz} \sin\theta) + e_{iy} (\sin\theta - Z_x \cos\theta)] D_1$$

Equation (12) now provides an expression for the local horizontally polarized incident electric field in terms of the two slopes of the plane, the angle of incidence and the components of the polarization vector.

To compute the local vertically polarized component, we will work with the magnetic field. The incident magnetic field ($\underline{H}_i$) can be calculated as follows:

$$\underline{H}_i = \frac{k_o \underline{n}_1 \times \underline{E}_i}{\omega\mu_o} \tag{13}$$

where $\omega = 2\pi f$ and f is frequency.
μ_o = magnetic permeability of free space.

The local vertically polarized component of the incident wave can be determined by computing $\underline{H}_i \cdot \underline{a}_y'$.

$$\underline{H}_i \cdot \underline{a}_y' = a_2 e^{-jk_o \underline{n}_1 \cdot \hat{\underline{r}}} \tag{14}$$

where

$$a_2 = \frac{k_o}{\omega\mu_o} [Z_y e_{iy} - (\sin\theta - Z_x \cos\theta)(e_{iz} \sin\theta + e_{ix} \cos\theta)] D_1$$

The components of the total electric field in the free space medium for the case of the local horizontally polarized incident wave were taken from ETL-7R-71-4 and are provided below:

$$E_{x'} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_{x1} (k_x, k_y) \exp (jk_x x' + jk_y y' - jk_z z') \, dk_x \, dk_y \tag{15}$$

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3 R. A. Hevenor, *Backscattering of Electromagnetic Waves from a Surface Composed of Two Types of Surface Roughness*, ETL-TR-71-4, U.S. Army Engineer Topographic Laboratories, Fort Belvoir, VA, October 1971, AD-737-675.

$$E_{y'} = a_1 \exp(-jk_o x' \sin\theta') \left[e^{jk_o z' \cos\theta'} + R_\perp e^{-jk_o z' \cos\theta'} \right] +$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_{y1}(k_x, k_y) \exp(jk_x x' + jk_y y' - jk_z z')\, dk_x\, dk_y \qquad (16)$$

$$E_z' = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_{z1}(k_x, k_y) \exp(jk_x x' + jk_y y' - jk_z z')\, dk_x\, dk_y \qquad (17)$$

where

$$k_z = \sqrt{k_o^2 - k_x^2 - k_y^2}$$

$$R_\perp = \frac{k_o \cos\theta' - \sqrt{k'^2 - k_o^2 \sin^2\theta'}}{k_o \cos\theta' + \sqrt{k'^2 - k_o^2 \sin^2\theta'}}$$

$$k' = \omega\sqrt{\mu_o \epsilon_o \epsilon_r}$$

$$A_{x1}(k_x, k_y) = a_1 k_x k_y Q'$$

$$A_{y1}(k_x, k_y) = -a_1 (k_x + k_z k_z') Q'$$

$$A_{z1}(k_x, k_y) = -a_1 k_y k_z' Q'$$

$$Q' = \frac{jT_\perp (k'^2 - k_o^2)\ S\ (k_x + k_o \sin\theta', k_y)}{2\pi (k_z + k_z')\ (k_x^2 + k_y^2 + k_z k_z')}$$

$$T_\perp = 1 + R_\perp$$

$$k_z' = \sqrt{k'^2 - k_x^2 - k_y^2}$$

$$S(k_x, k_y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} s(x', y') e^{-jk_x x'} e^{-jk_y y'} dy' dx'$$

The definition of the various parameters given above are provided in the back of this report. The terms with the double integrals in equations (15) through (17) represent the scattered fields, however, the term in equation (16) without double integrals represents an incident wave plus a specular component. The parameter $s(x', y')$ is the expression for the slightly rough surface in the primed or local coordinate system.

The components of the total electric field in the free space medium for the case of the local vertically polarized incident wave are taken from ETL-TR-71-4 and given as follows:[4]

$$E_{x'} = a_2 \eta e^{-jk_o x' \sin\theta'} \cos\theta' \left\{ R_{\parallel} e^{-jk_o z' \cos\theta'} - e^{jk_o z' \cos\theta'} \right\}$$

$$+ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A_{x2}(k_x, k_y) \exp(jk_x x' + jk_y y' - jk_z z')\, dk_x\, dk_y \tag{18}$$

$$E_{y'} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A_{y2}(k_x, k_y) \exp(jk_x x' + jk_y y' - jk_z z')\, dk_x\, dk_y \tag{19}$$

$$E_{z'} = -a_2 \eta e^{-jk_o x' \sin\theta'} \sin\theta' \left\{ e^{jk_o z' \cos\theta'} + R_{\parallel} e^{-jk_o z' \cos\theta'} \right\}$$

$$+ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A_{z2}(k_x, k_y) \exp(jk_x x' + jk_y y' - jk_z z')\, dk_x\, dk_y \tag{20}$$

where

$$R_{\parallel} = \frac{k'^2 \cos\theta' - k_o \sqrt{k'^2 - k_o^2 \sin^2\theta'}}{k'^2 \cos\theta' + k_o \sqrt{k'^2 - k_o^2 \sin^2\theta'}}$$

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4 R. A. Hevenor, *Backscattering of Electromagnetic Waves from a Surface Composed of Two Types of Surface Roughness*, ETL-TR-71-4, U.S. Army Engineer Topographic Laboratories, Fort Belvoir, VA, October 1971, AD-737-675.

$$\eta = c\mu_o$$

$$A_{x2}(k_x, k_y) = \frac{a_2 \eta}{k_o} \left\{ k_z G_y(k_x, k_y) + k_y F_z(k_x, k_y) \right\}$$

$$A_{y2}(k_x, k_y) = -\frac{a_2 \eta}{k_o} \left\{ k_z D_x(k_x, k_y) + k_x F_z(k_x, k_y) \right\}.$$

$$A_{z2}(k_x, k_y) = \frac{a_2 \eta}{k_o} \left\{ k_x G_y(k_x, k_y) - k_y D_x(k_x, k_y) \right\}$$

The three quantities $D_x(k_x, k_y)$, $G_y(k_x, k_y)$, and $F_z(k_x, k_y)$ are derived in detail up to first order in perturbation in ETL-TR-71-4 and are stated below:[5]

$$D_x(k_x, k_y) = \frac{B_o a_{11} - A_o a_{21}}{a_{11} a_{22} - a_{12} a_{21}}$$

$$G_y(k_x, k_y) = \frac{A_o a_{22} - B_o a_{12}}{a_{11} a_{22} - a_{12} a_{21}}$$

$$F_z(k_x, k_y) = \frac{k_y G_y(k_x, k_y) + k_x D_x(k_x, k_y)}{k_z}$$

where

$$A_o = -\left\{ \eta k_o^2 k_y^2 k_z Q_1 + \eta k_z k_o^2 k_z'^2 Q_1 + k_z k_z' k_o k'^2 W + k_z k_z' k_o k'^2 V \right\}$$

$$B_o = k_z k_z' k_o k'^2 S_o - \eta k_o^2 k_x k_y k_z Q_1$$

$$a_{11} = \eta k_y^2 k_z' k'^2 + \eta k_z^2 k_z' k'^2 + \eta k_o^2 k_y^2 k_z + \eta k_o^2 k_z'^2 k_z$$

$$a_{12} = \eta k_y k_z' k'^2 k_x + \eta k_o^2 k_y k_z k_x$$

$$a_{21} = \eta k'^2 k_x k_z' k_y + \eta k_o^2 k_x k_z k_y$$

5 R. A. Hevenor, *Backscattering of Electromagnetic Waves from a Surface Composed of Two Types of Surface Roughness*, ETL-TR-71-4, U.S. Army Engineer Topographic Laboratories, Fort Belvoir, VA, October 1971, AD-737-675.

$$a_{22} = \eta \, [k'^2 \, k_z^2 \, k_z' + k'^2 \, k_x^2 \, k_z' + k_o^2 \, k_z \, k_z'^2 + k_o^2 \, k_x^2 \, k_z]$$

$$Q_1 = \frac{j\,(k_o^2 - k'^2)\,(1 - R_{||})\cos\theta'\,S\,(k_x + k_o\sin\theta', k_y)}{2\pi k_o}$$

$$V = -\,j\,T_{||}\,\eta\,k_o\sin^2\theta'\,(k_o^2 - k'^2)\,S\,(k_x + k_o\sin\theta', k_y)\,/\,(2\pi\,k'^2)$$

$$W = j\,(k_x + k_o\sin\theta')\,\eta\,T_{||}\sin\theta'\,(k_o^2 - k'^2)\,S\,(k_x + k_o\sin\theta', k_y)\,/\,(2\pi\,k'^2)$$

$$S_o = j\,k_y\,\eta\,T_{||}\sin\theta'\,(k_o^2 - k'^2)\,S\,(k_x + k_o\sin\theta', k_y)\,/\,(2\pi\,k'^2)$$

$$T_{||} = 1 + R_{||}$$

$$S\,(k_x, k_y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s\,(x', y')\, e^{-jk_x x'}\, e^{-jk_y y'}\, dx'\,dy'$$

The parameter $R_{||}$ is the Fresnel reflection coefficient for a vertically polarized wave. The backscattered far field ($\underline{E}_s$) can be calculated by evaluating the scattered fields using the method of stationary phase. This approach is given by Collin and Zucker.[6] The same result can be obtained by placing the total fields given by equations (15) through (20) into the Stratton-Chu integral and evaluating it.

$$\underline{E}_s = (2\pi)^2\, 2\cos\theta'\, \frac{(jk_o)\, e^{-jk_oR}}{4\pi R} \left\{ \underline{a}_x'\, A_x\,(k_o\sin\theta', 0) + \underline{a}_y'\, A_y\,(k_o\sin\theta', 0) + \underline{a}_z'\, A_z\,(k_o\sin\theta', 0) \right\} \tag{21}$$

where

$$A_x\,(k_o\sin\theta', 0) = A_{x1}\,(k_o\sin\theta', 0) + A_{x2}\,(k_o\sin\theta', 0) \tag{22}$$

$$A_y\,(k_o\sin\theta', 0) = A_{y1}\,(k_o\sin\theta', 0) + A_{y2}\,(k_o\sin\theta', 0) \tag{23}$$

$$A_z\,(k_o\sin\theta', 0) = A_{z1}\,(k_o\sin\theta', 0) + A_{z2}\,(k_o\sin\theta', 0) \tag{24}$$

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[6] Robert E. Collin and Francis J. Zucker, *Antenna Theory Part I*, New York, McGraw-Hill Book Co., 1969.

The parameter R in equation (21) is simply the distance from the origin of the local coordinate system (x'y'z') to the receiver in the far field. Equations (22) through (24) can easily be evaluated using previous results.

$$A_{x1}\,(k_o \sin\theta', 0) = 0 \tag{25}$$

$$A_{x2}\,(k_o \sin\theta', 0) = a_2\, \eta\, A \cos\theta'\, S\,(2\,k_o \sin\theta', 0) \tag{26}$$

where

$$A = -\left\{k_z\, k_o^2\, k_z'^2\, C_o + k_z\, k_z'\, k_o\, k'^2\, C_1 + k_z\, k_z'\, k_o\, k'^2\, C_2\right\} / \,(k_z^2\, k_z'\, k'^2 + k_o^2\, k_z'^2 k_z)$$

$$C_o = j\,(k_o^2 - k'^2)\,(1 - R_{\parallel})\cos\theta'/(2\pi\, k_o)$$

$$C_1 = j\,(2k_o \sin^2\theta')\, T_{\parallel}\, (k_o^2 - k'^2)/(2\pi\, k'^2)$$

$$C_2 = -j\, T_{\parallel}\, k_o \sin^2\theta'\,(k_o^2 - k'^2)/(2\pi\, k'^2)$$

$$A_{y1}\,(k_o \sin\theta', 0) = j\, a_1\, T_{\perp}\,(k_o^2 - k'^2)\, S\,(2k_o \sin\theta', 0)/[2\pi\,(k_z + k_z')] \tag{27}$$

$$A_{y2}\,(k_o \sin\theta', 0) = 0 \tag{28}$$

$$A_{z1}\,(k_o \sin\theta', 0) = 0 \tag{29}$$

$$A_{z2}\,(k_o \sin\theta', 0) = a_2\, \eta\, A \sin\theta'\, S\,(2k_o \sin\theta', 0) \tag{30}$$

The backscattered far field can now be written as follows:

$$\underline{E}_s = (2\pi)^2\; 2\cos\theta'\; \frac{(jk_o)\, e^{-jk_oR}}{4\pi R} \left\{\underline{a}_x'\, A_{11} + \underline{a}_y'\, A_{22} + \underline{a}_z'\, A_{33}\right\} S\,(2k_o \sin\theta', 0) \tag{31}$$

where

$$A_{11} = a_2\, \eta\, A \cos\theta'$$

$$A_{22} = j\, a_1\, T_{\perp}\,(k_o^2 - k'^2)/[2\pi\,(k_z + k_z')]$$

$$A_{33} = a_2\, \eta\, A \sin\theta'$$

By using previously derived results, one can now write $\underline{E}_s$ in terms of the unit vectors of the original reference coordinate system (x, y, z).

$$\underline{E}_s = (2\pi)^2 \, 2\cos\theta' \, \frac{(jk_o)\, e^{-jk_oR}}{4\pi R} \left\{ \hat{A}_x \underline{a}_x + \hat{A}_y \underline{a}_y + \hat{A}_z \underline{a}_z \right\} S(2k_o \sin\theta', 0) \qquad (32)$$

$$\hat{A}_x = A_{11}\,\alpha_{x1} + A_{22}\,\alpha_{y1} + A_{33}\,\alpha_{z1}$$

$$\hat{A}_y = A_{11}\,\alpha_{x2} + A_{22}\,\alpha_{y2} + A_{33}\,\alpha_{z2}$$

$$\hat{A}_z = A_{11}\,\alpha_{x3} + A_{22}\,\alpha_{y3} + A_{33}\,\alpha_{z3}$$

$$\underline{a}'_x = \alpha_{x1}\,\underline{a}_x + \alpha_{x2}\,\underline{a}_y + \alpha_{x3}\,\underline{a}_z$$

$$\alpha_{x1} = (\sin\theta - Z_x\cos\theta + Z_y^2\sin\theta)\, D_o\, D_1$$

$$\alpha_{x2} = -\,Z_y\,(\cos\theta + Z_x\sin\theta)\, D_o\, D_1$$

$$\alpha_{x3} = [Z_x\,(\sin\theta - Z_x\cos\theta) - Z_y^2\cos\theta]\, D_o\, D_1$$

$$\underline{a}'_y = \alpha_{y1}\,\underline{a}_x + \alpha_{y2}\,\underline{a}_y + \alpha_{y3}\,\underline{a}_z$$

$$\alpha_{y1} = Z_y\, D_1\cos\theta$$

$$\alpha_{y2} = (\sin\theta - Z_x\cos\theta)\, D_1$$

$$\alpha_{y3} = Z_y\, D_1\sin\theta$$

$$\underline{a}'_z = \alpha_{z1}\,\underline{a}_x + \alpha_{z2}\,\underline{a}_y + \alpha_{z3}\,\underline{a}_z$$

$$\alpha_{z1} = -\,Z_x\, D_o$$

$$\alpha_{z2} = -\,Z_y\, D_o$$

$$\alpha_{z3} = D_o$$

If the radar receiver is sensitive to an electric field, which has a unit polarization vector $\underline{e}_r$, then the received field (E_R) will be

$$E_R = \underline{e}_r \cdot \underline{E}_s \qquad (33)$$

The unit polarization vector $\underline{e}_r$ will be allowed to have complex components in all three dimensions.

$$\underline{e}_r = e_{rx}\,\underline{a}_x + e_{ry}\,\underline{a}_y + e_{rz}\,\underline{a}_z$$

The received electric field can now be written as

$$E_r = 2\,(2\pi)^2 \cos\theta' \frac{(jk_o)\,e^{-jk_oR}}{4\pi R} A_{pp}\, S\,(2k_o \sin\theta', 0) \tag{34}$$

where

$$A_{pp} = \hat{A}_x\, e_{rx} + \hat{A}_y\, e_{ry} + \hat{A}_z\, e_{rz}$$

The radar backscatter coefficient σ^o can be computed from the received electric field as follows:

$$\sigma^o = \frac{4\pi R^2}{A_o} \frac{<E_r\, E_r^*>}{\underline{E}_i \cdot \underline{E}_i^*} \tag{35}$$

The quantity A_o is the area of the surface S which is illuminated by the radar beam. The brackets around $E_r E_r^*$ are used to indicate the calculation of a statistical average. The random portion of E_r comes from $S(2k_o \sin\theta', 0)$ since the surface function $s\,(x', y')$ is random.

$$<E_r\, E_r^*> = \frac{4\pi^2\, k_o^2 \cos^2\theta'}{R^2} A_{pp}\, A_{pp}^* <S\,(2k_o \sin\theta', 0)\, S^*\,(2k_o \sin\theta', 0)>$$

It remains now to calculate the statistical average indicated in the above equation.

$$<S\,(k_x, k_y)\, S^*\,(k_x, k_y)> = <S\,(k_x, k_y)\, S\,(-k_x, -k_y)> = \frac{1}{4\pi^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}$$

$$<s\,(x', y')\, s\,(x'', y'')>\, e^{-jk_x(x'-x'')}\, e^{-jk_y(y'-y'')}\, dy'\, dx'\, dy''\, dx''$$

If we let $\tau_x = x' - x''$ and $\tau_y = y' - y''$ and we realize that $<s\,(x', y')\; s\,(x'', y'')>$ is the correlation function of the surface, then we have

$$< S(k_x, k_y)\, S(-k_x, -k_y) > = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dy' \int_{-\infty}^{\infty} d\tau_x \int_{-\infty}^{\infty} d\tau_y \; x$$

$$\phi(\tau_x, \tau_y)\, e^{-jk_x \tau_x}\, e^{-jk_y \tau_y}$$

The integrals in x′ and y′ at first appear to be meaningless; however, the result of the two integrals can be represented approximately by the illuminated area A_o, as long as the dimensions of this area are much greater than the correlation distances associated with $\phi(\tau_x, \tau_y)$. The other two integrals in τ_x and τ_y can be used to define the surface roughness spectrum, $W(k_x, k_y)$:

$$W(k_x, k_y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\tau_x \int_{-\infty}^{\infty} d\tau_y \, C(\tau_x, \tau_y)\, e^{-jk_x \tau_x}\, e^{-jk_y \tau_y}$$

where

$$\phi(\tau_x, \tau_y) = \sigma_1^2 \, C(\tau_x, \tau_y)$$

The result for the needed average can now be written as

$$< S(k_x', k_y)\, S^*(k_x, k_y) > = \frac{A_o \sigma_1^2}{2\pi} \, W(k_x, k_y)$$

The radar backscatter coefficient can now be written in its final form:

$$\sigma^o = 8\pi^2 k_o^2 \sigma_1^2 \cos^2 \theta' A_{pp} A_{pp}^* \, W(2k_o \sin\theta', 0) \tag{36}$$

This answer is a function of two slopes (Z_x and Z_y) and the surface roughness spectrum. In the next section, we will pick a specific form for the correlation function, and the roughness spectrum will be computed. This in turn will allow us to calculate σ^o as a function of the incidence angle, θ. Before we do this, however, two special cases of slope must be considered separately so that computational difficulties will not arise. The first special case occurs when $Z_x = 0$ and $Z_y = 0$. It can be seen from the equation for $\underline{a}_y'$ (equation 7) that at $\theta = 0°$ we will encounter a zero over zero condition when both slopes vanish. Therefore, for this case we will need to calculate new values for $\underline{a}_x'$, $\underline{a}_y'$, $\underline{a}_z'$, a_1, and a_2. In this special case, the unit vectors $\underline{a}_z'$ and $\underline{n}$ are both equal to $\underline{a}_z$. The quantity $\underline{a}_y'$ then becomes

$$\underline{a}_y' = \underline{a}_y$$

This makes $\underline{a}_x'$ equal to $\underline{a}_x$. The parameters a_1 and a_2 can be calculated easily:

$$a_1 = \underline{a}_y' \cdot \left\{ e_{ix}\, \underline{a}_x + e_{iy}\, \underline{a}_y + e_{iz}\, \underline{a}_z \right\} = e_{iy}$$

$$a_2 = - \frac{k_o}{\omega\mu_o} (e_{iz} \sin\theta + e_{ix} \cos\theta)$$

The second special case occurs when $Z_y = 0$ and $Z_x = \tan\theta$. It can be seen from equation 7 that this condition again yields an indeterminant zero over zero situation. We will define a unit vector $\underline{n}_3$ that will be used to calculate $\underline{a}_y'$:

$$\underline{n}_3 = \underline{a}_x \cos\theta + \underline{a}_z \sin\theta$$

The quantity $\underline{a}_y'$ will now be defined in the following expression for this special case:

$$\underline{a}_y' = \frac{\underline{n} \times \underline{n}_3}{|\underline{n} \times \underline{n}_3|}$$

$$\underline{n} \times \underline{n}_3 = \underline{a}_y$$

$$\underline{a}_y' = \underline{a}_y$$

Since $\underline{a}_z'$ is equivalent to $\underline{n}$, we can now calculate $\underline{a}_x'$ for this special case.

$$\underline{a}_x' = \frac{\underline{a}_x + \underline{a}_z \tan\theta}{\sqrt{1 + \tan^2\theta}}$$

The only remaining computation that needs to be performed is to determine a_1 and a_2:

$$a_1 = \underline{a}_y' \cdot \left\{ e_{ix}\, \underline{a}_x + e_{iy}\, \underline{a}_y + e_{iz}\, \underline{a}_z \right\} = e_{iy}$$

$$a_2 = - \frac{k_o}{\omega\mu_o} (e_{iz} \sin\theta + e_{ix} \cos\theta)$$

It can now be seen that the solutions for a_1 and a_2 for this special case are identical to the solutions for a_1 and a_2 for the first special case given previously, which was not obvious. This completes the derivation of the radar backscatter coefficient as a function of the two slopes Z_x and Z_y. The final result does not consider depolarization due to the slightly rough surface itself, since only first order terms in perturbation were

used. However, depolarization due to the tilted plane is considered. In the next section, sample calculations will be made for the specific case where $C(\tau_x, \tau_y)$ has a gaussian form.

RESULTS

The purpose of this section is to evaluate the radar backscatter coefficient (equation 36) for a particular surface correlation function. There are two correlation functions that are used most often in theoretical scattering work, and these are the exponential and the gaussian functions. If we use a gaussian correlation function, then $C(\tau_x, \tau_y)$ will have the following form:

$$C(\tau_x, \tau_y) = e^{-(\tau_x^2 + \tau_y^2)/\ell^2} \tag{37}$$

where ℓ is the correlation distance.

In using the above expression for $C(\tau_x, \tau_y)$, we have assumed an isotropic correlation function. This assumption would not be valid for surfaces that contain some type of periodic structure in one direction, i.e. a plowed field. Recalling the definition of the surface roughness spectrum $W(k_x, k_y)$ and inserting equation 37 into that expression will yield

$$W(k_x, k_y) = \frac{\ell^2}{2} e^{-(k_x^2 + k_y^2)\ell^2/4}$$

A computer program was written for the calculation of equation 36 using the surface roughness spectrum given above. The computer program listing for a vertical polarization solution is provided in the appendix. A set of numerical results for four different polarizations is given in figures 2 through 13. Each figure presents two graphs of radar backscatter coefficient in decibels versus angle of incidence in degrees. The backscatter coefficient in decibels is related to equation 36 as follows:

$$\sigma^\circ \text{ (in decibels)} = 10 \log_{10} \sigma^\circ$$

The backscatter coefficient on the right side of the above equation is computed from equation 36. The numerical values used for frequency, correlation length, standard deviation, and relative dielectric constant were held fixed throughout all calculations and are given on each figure.

The four polarizations considered were horizontal, vertical, circular, and elliptical. The unit polarization vectors for each of the four cases are given below:

For horizontal polarization $\quad \underline{e}_i = \underline{e}_r = \underline{a}_y$

For vertical polarization $\underline{e}_i = \underline{e}_r = -(\underline{a}_x \cos\theta + \underline{a}_z \sin\theta)$

For circular polarization $\underline{e}_i = \frac{1}{\sqrt{2}}\left\{\underline{a}_y - e^{-j\pi/2}(\underline{a}_x \cos\theta + \underline{a}_z \sin\theta)\right\}$

For circular polarization $\underline{e}_r = \frac{1}{\sqrt{2}}\left\{\underline{a}_y - e^{j\pi/2}(\underline{a}_x \cos\theta + \underline{a}_z \sin\theta)\right\}$

For elliptical polarization $\underline{e}_i = \frac{1}{\sqrt{2}}\left\{\underline{a}_y - e^{-j\pi/4}(\underline{a}_x \cos\theta + \underline{a}_z \sin\theta)\right\}$

For elliptical polarization $\underline{e}_r = \frac{1}{\sqrt{2}}\left\{\underline{a}_y - e^{j\pi/4}(\underline{a}_x \cos\theta + \underline{a}_z \sin\theta)\right\}$

There are three figures associated with each polarization. One figure shows the effect of changing only the slope in the plane of incidence (Z_x). The next figure shows the effect of changing only the slope in the plane orthogonal to the plane of incidence (Z_y). The third figure shows the effect of having an equal slope in both the x and y directions. For all figures, a curve is plotted for the case where no slopes exist ($Z_x = 0$, $Z_y = 0$). The results for each polarization will be discussed separately below.

The horizontal polarization results are presented in figures 2 through 4. By comparing figures 2 and 3, it can be seen that Z_x influences horizontal polarization much more than Z_y. The value of slope, which is used in all calculations, is equal to 1.2. In figure 2, adding a large slope in x lowers the value of $\sigma°$ over a range of incidence angles extending from 0° to approximately 25°. For angles of incidence greater than 25°, the $\sigma°$ curve for zero slope drops off much faster than the curve associated with a slope in x. The crossover point in figure 2 occurs at the point where $\theta' = \theta$. The crossover point in figures 3 and 4 are more difficult to interpret since both local vertical and local horizontal polarization components exist. Figure 4 shows the result of computing the backscatter coefficient for a surface that has a large slope in both x and y.

The vertical polarization results are given in figures 5 through 7. By comparing figures 5 and 6, one sees that Z_y has a much greater influence on $\sigma°$ than does Z_x. This influence, however, is almost a bias as we see that the two curves in figure 6 have essentially the same shape. Figure 7 presents the results for a surface that has slopes in both x and y. In this figure, we see that making $Z_x = 1.2$ and $Z_y = 1.2$ lowers the curve over the range of incidence angles extending from 0° to approximately 63°.

The results of the circular polarization calculations are presented in figures 8 through 10. Figure 8 shows the influence of a large slope in x and no slope in y. It can be seen that again we have a crossover point at approximately $\theta = 25°$. Figure 9 shows the influence of a large slope in y and no slope in x. The $\sigma°$ curve for $Z_y = 1.2$

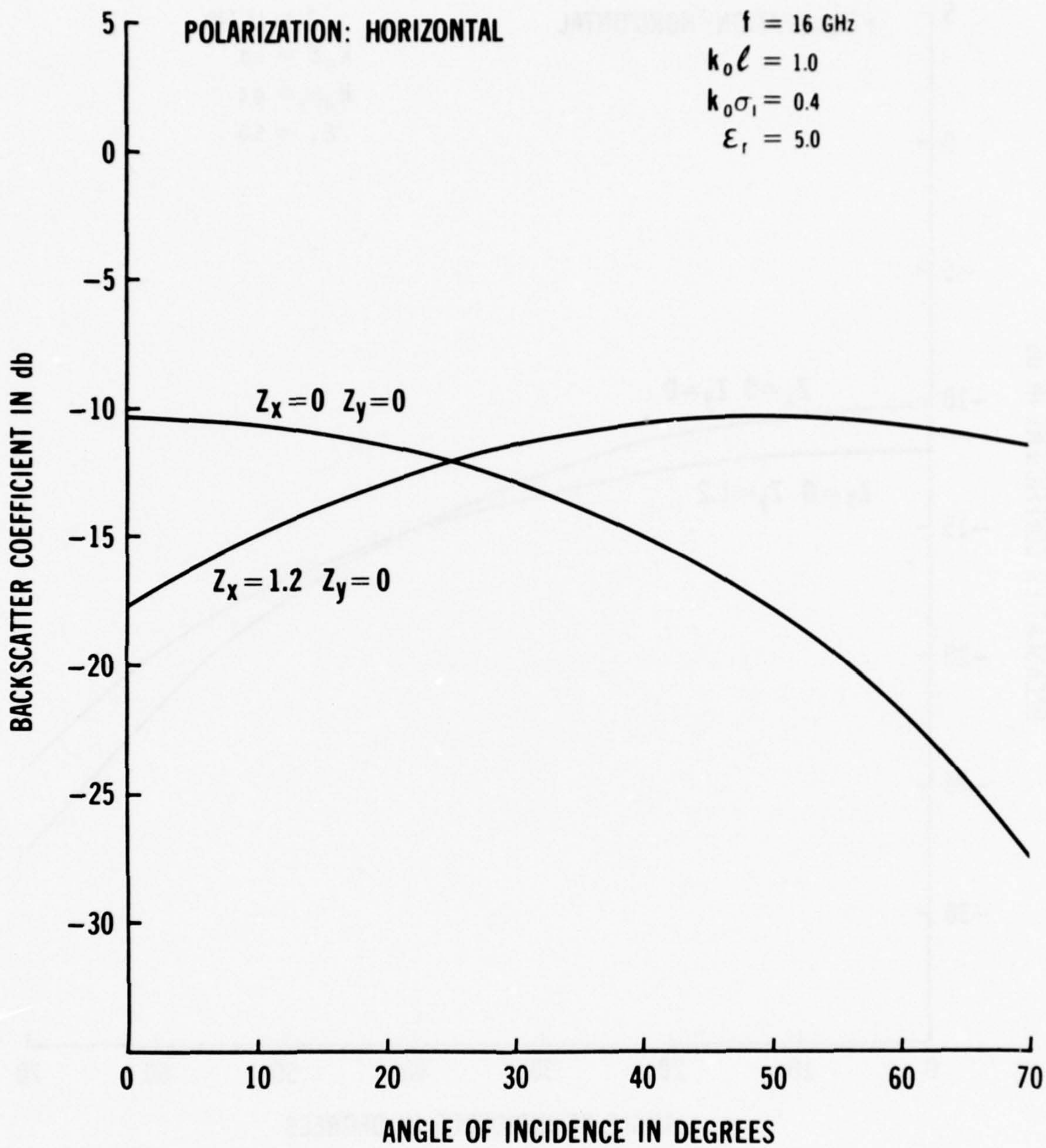

Figure 2. Study of Z_x Variations

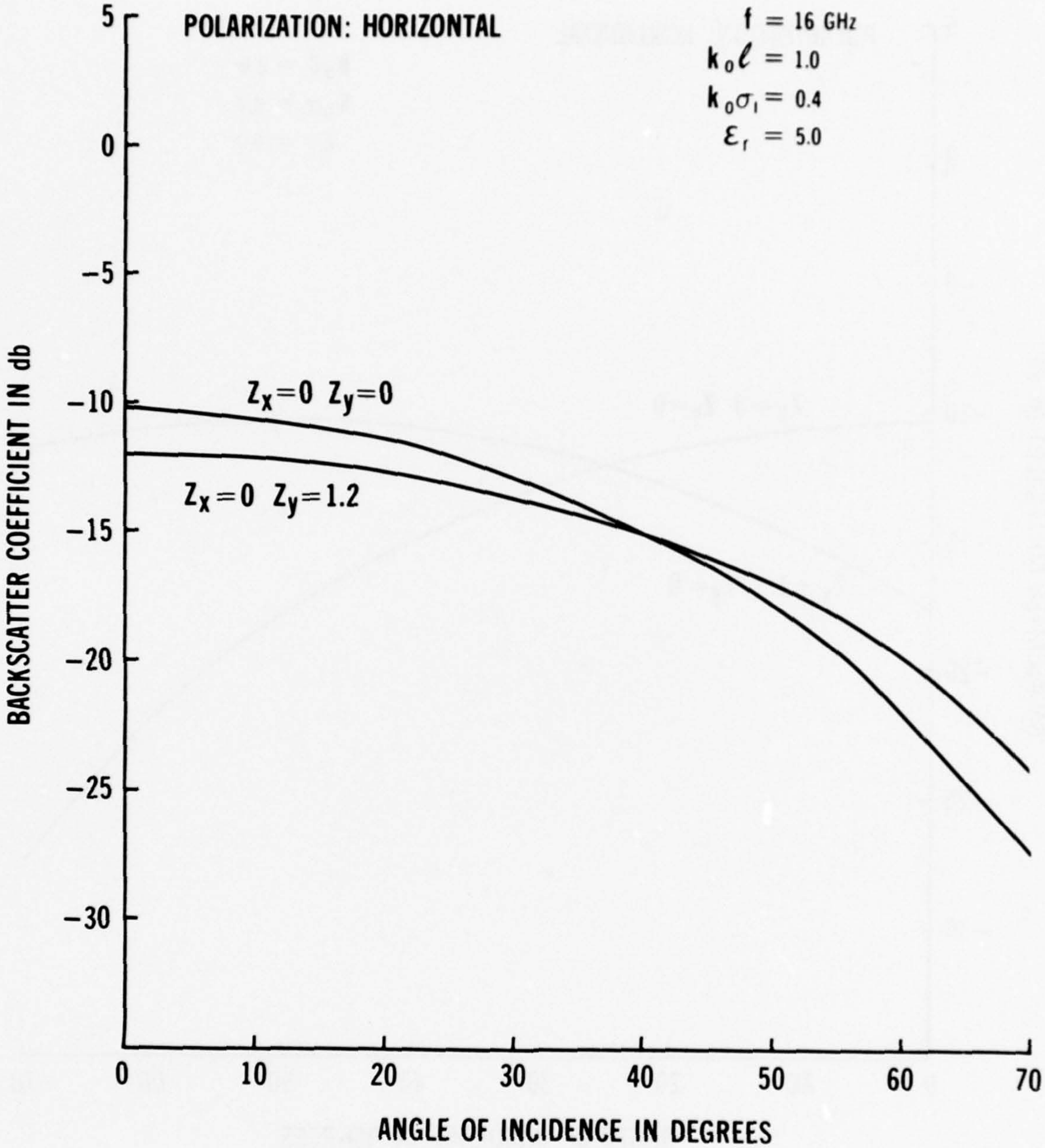

Figure 3. Study of Z_y Variations

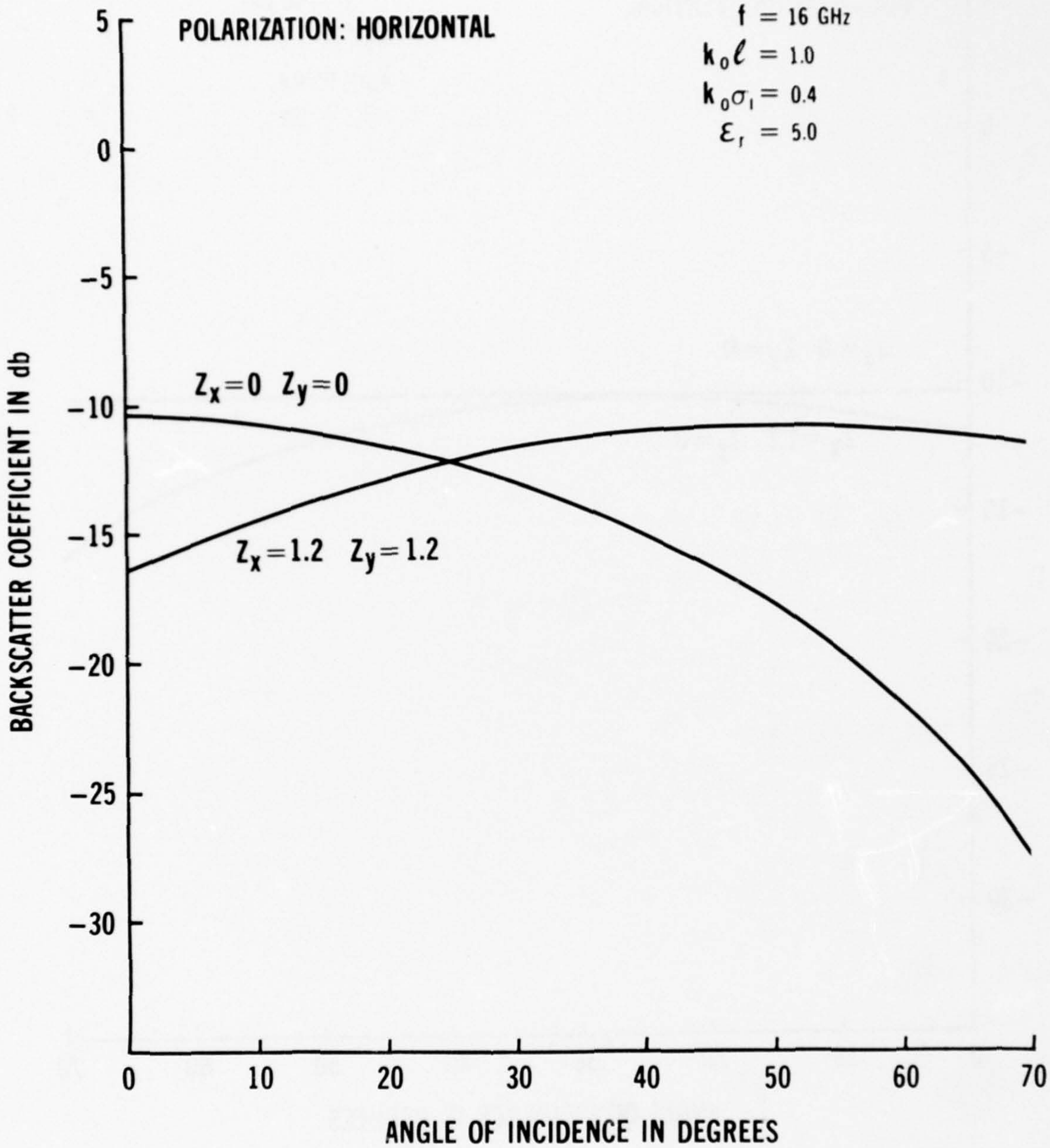

Figure 4. Study of Z_x and Z_y Variations

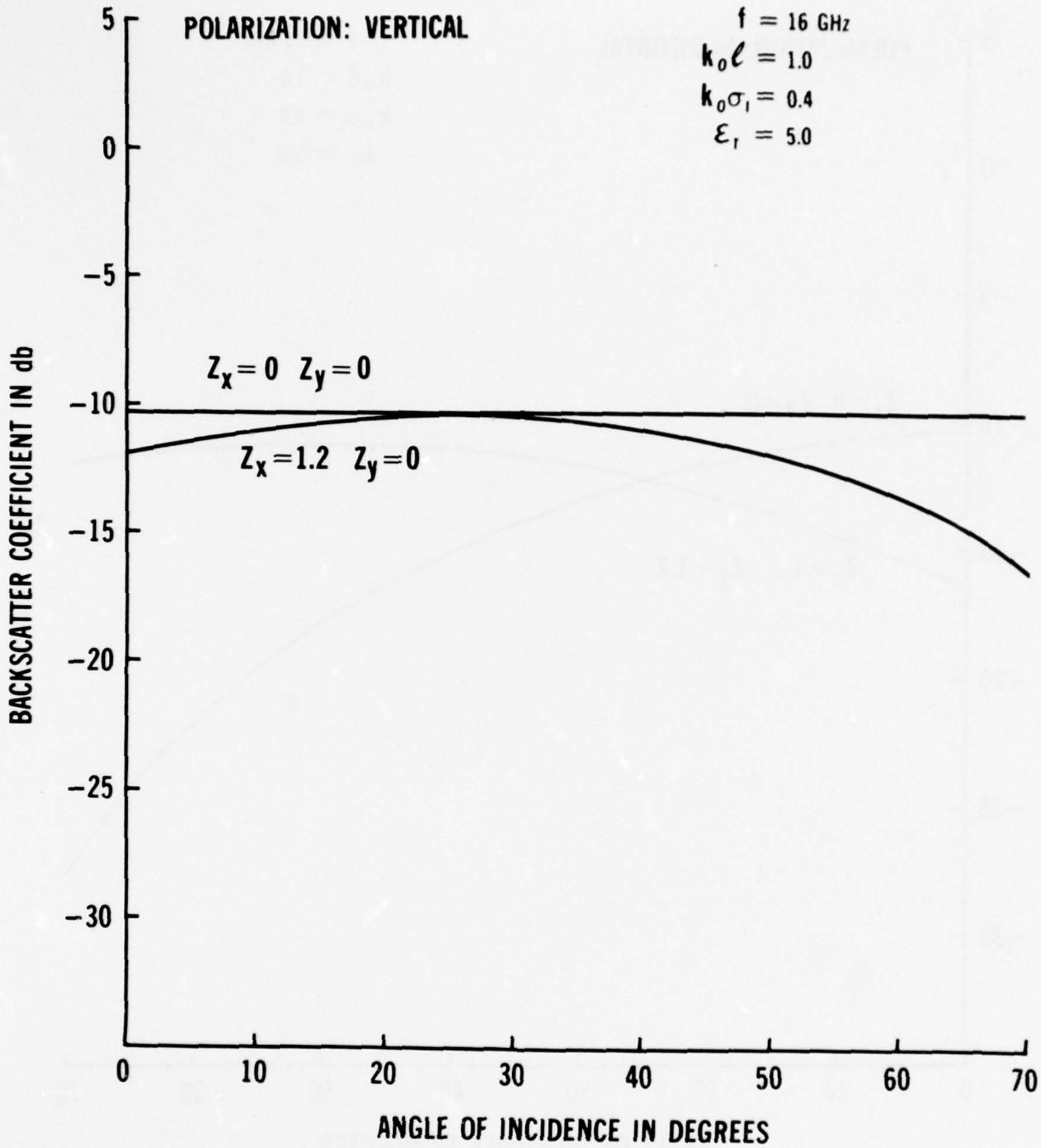

Figure 5. Study of Z_x Variations

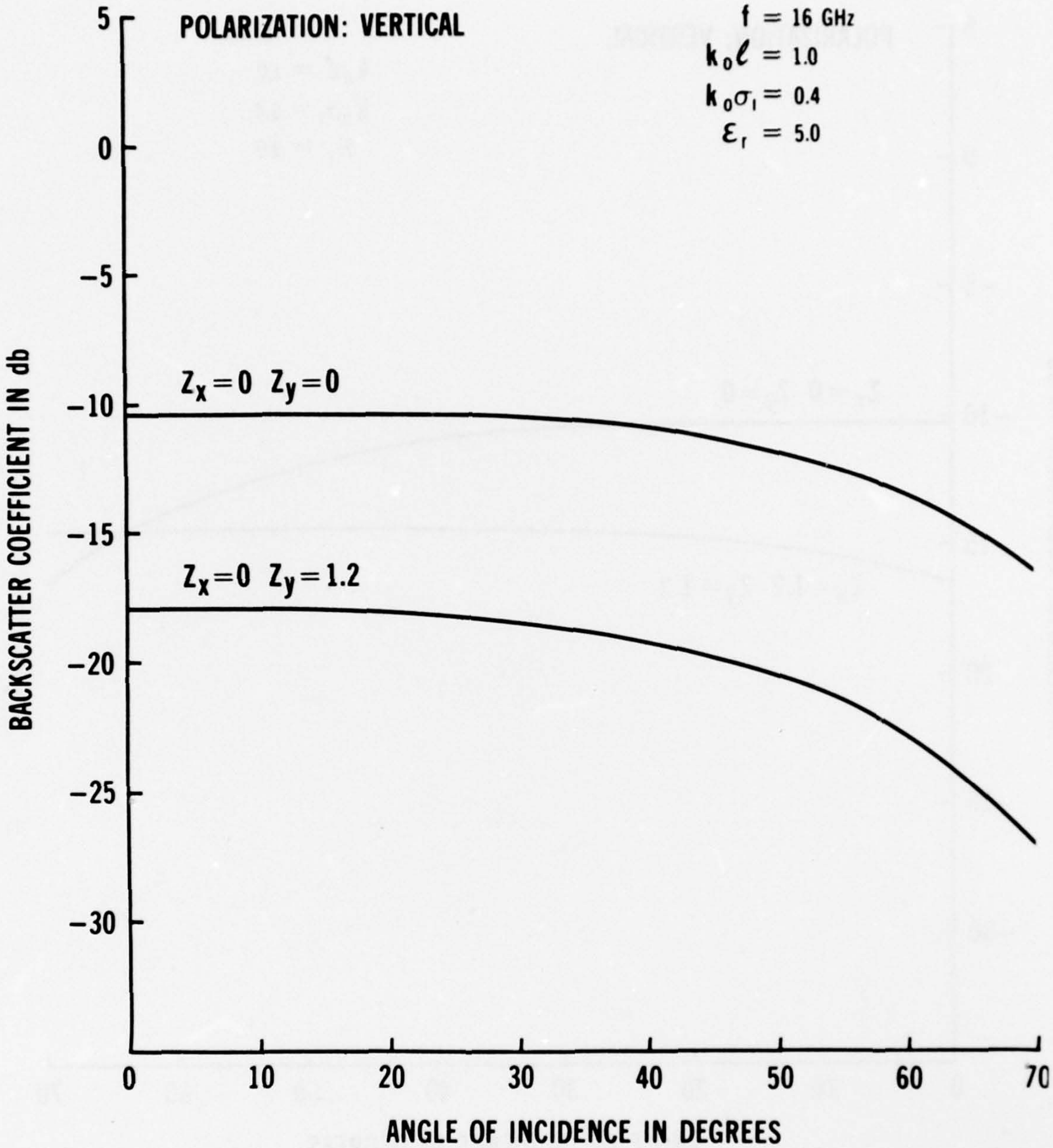

Figure 6. Study of Z_y Variations

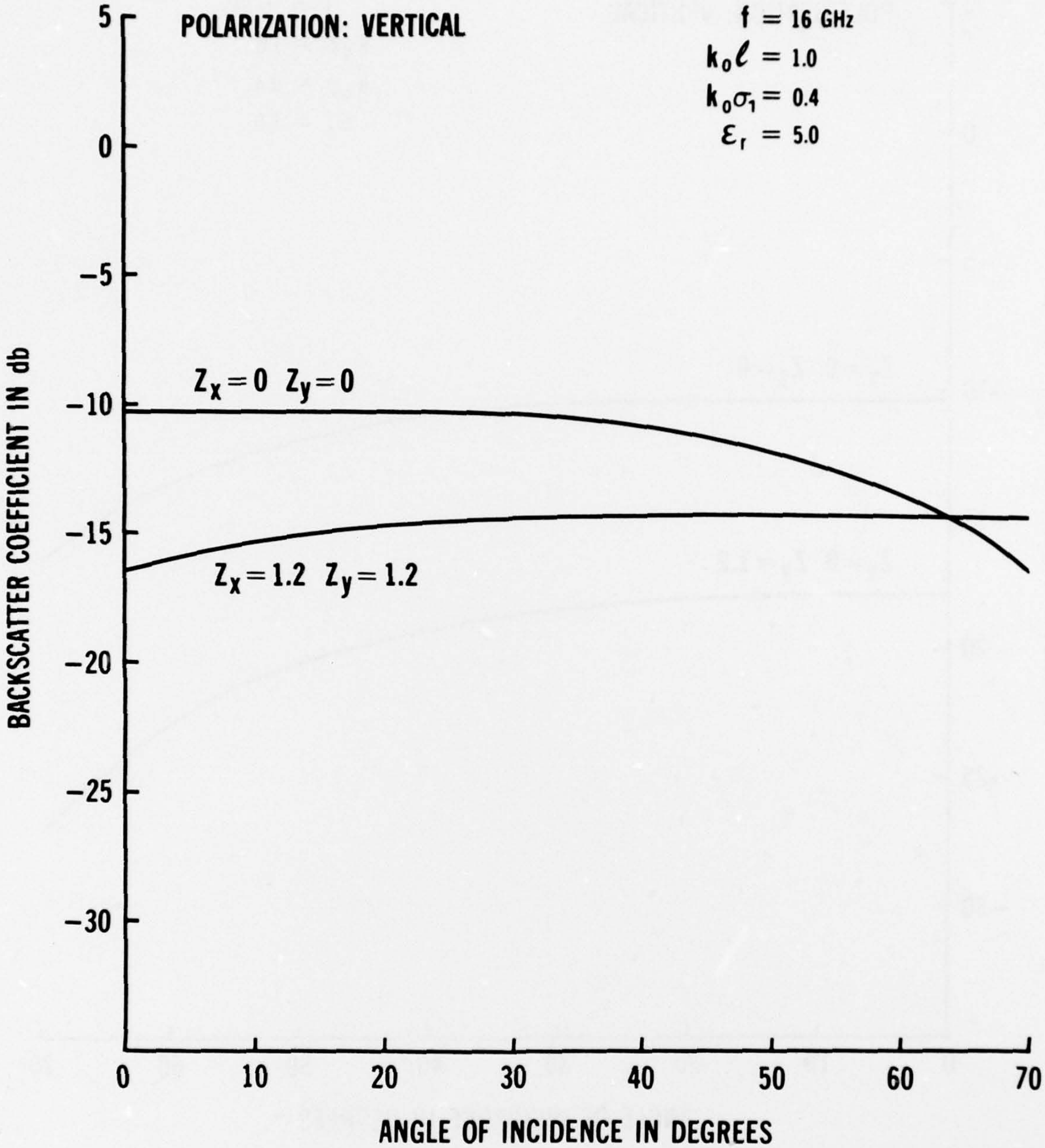

Figure 7. Study of Z_x and Z_y Variations

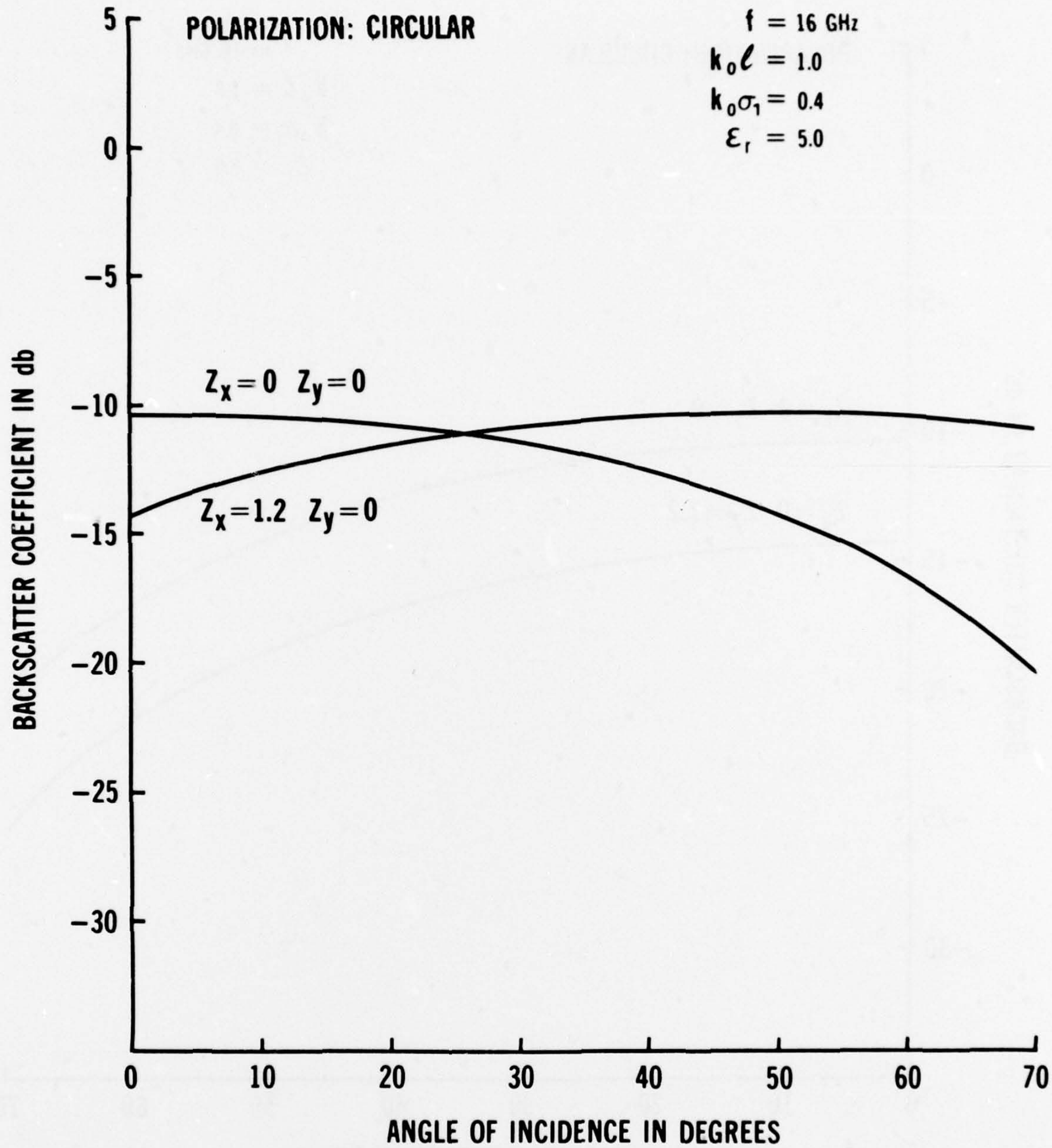

Figure 8. Study of Z_x Variations

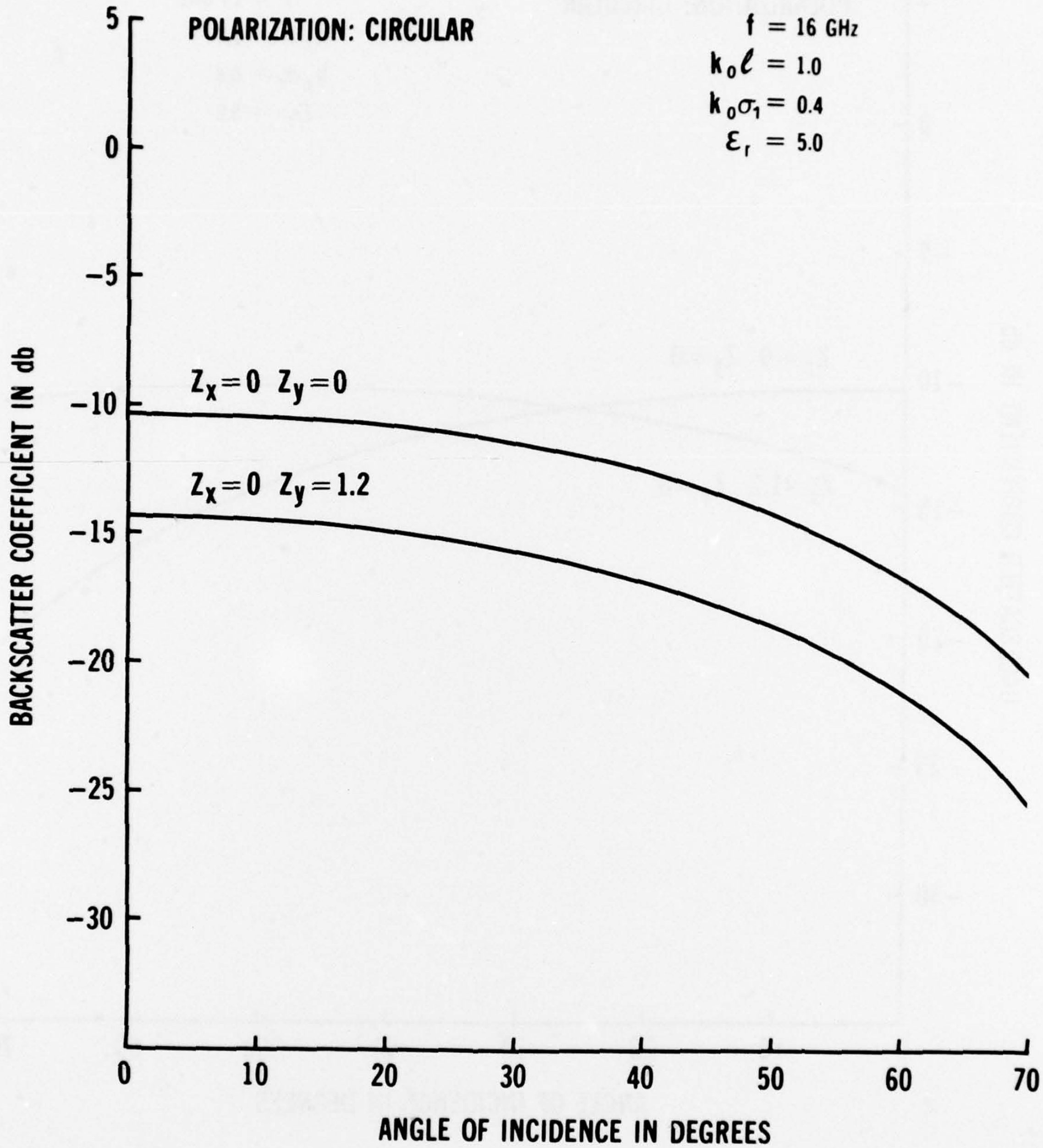

Figure 9. Study of Z_y Variations

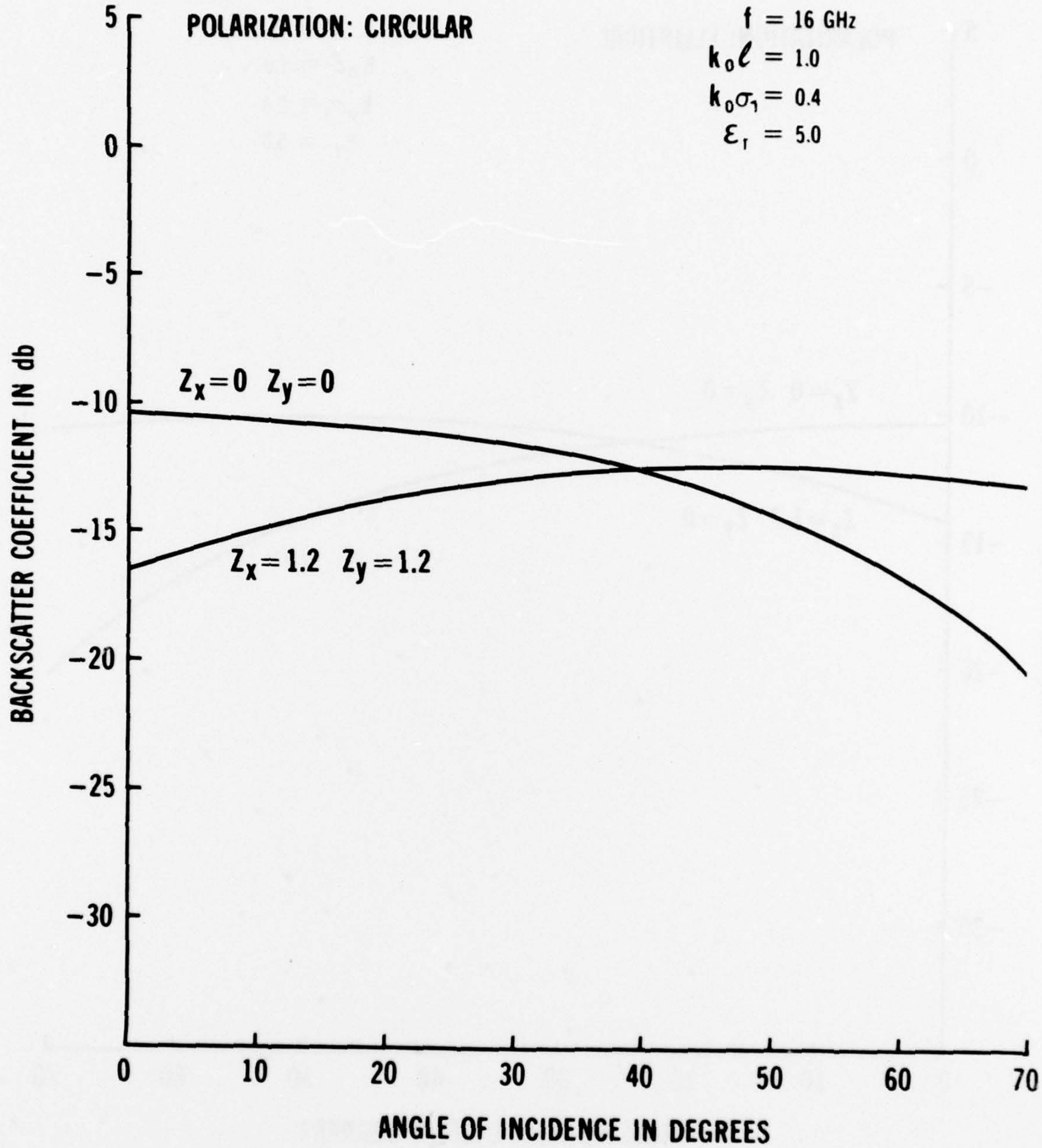

Figure 10. Study of Z_x and Z_y Variations

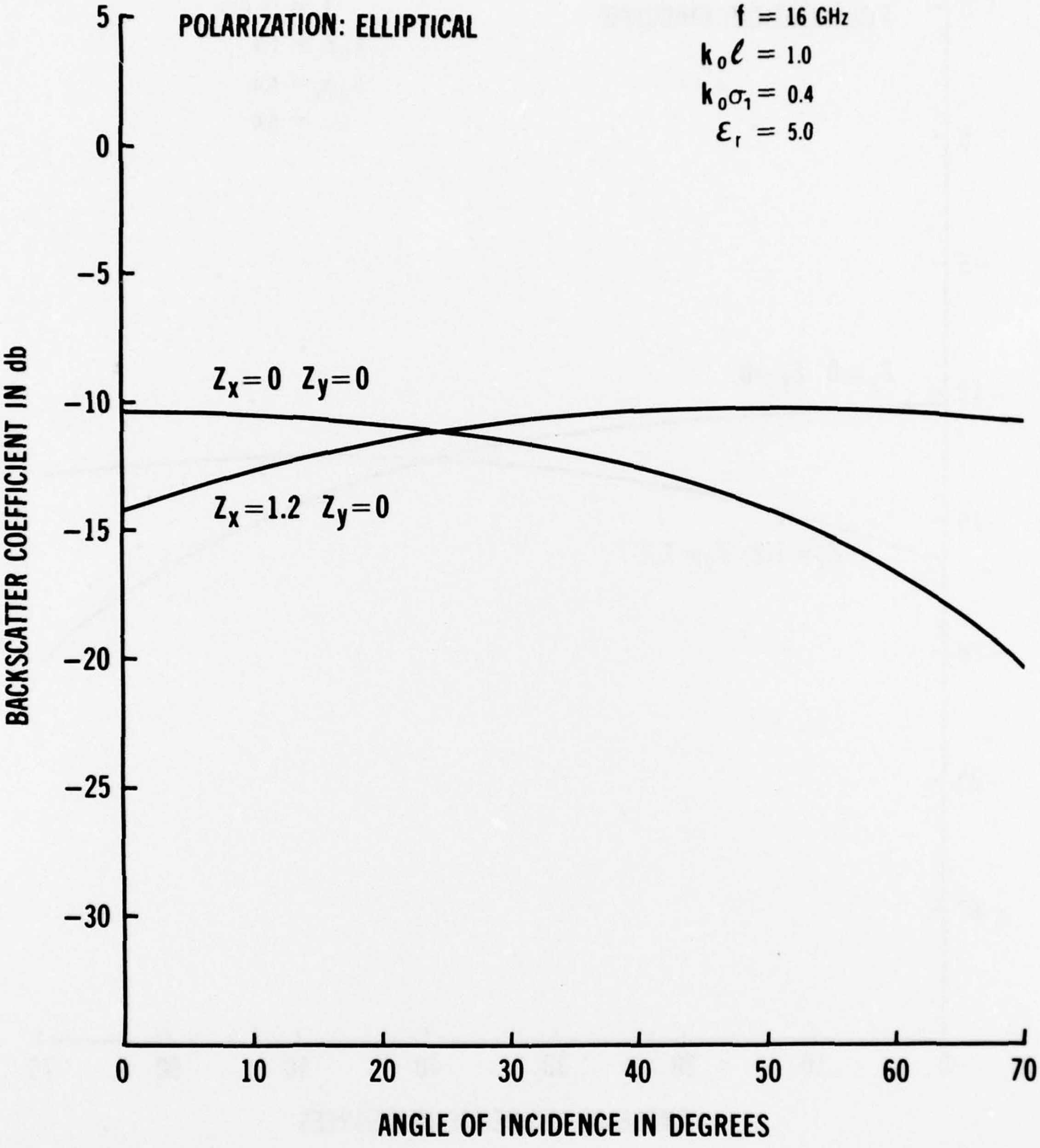

Figure 11. Study of Z_x Variations

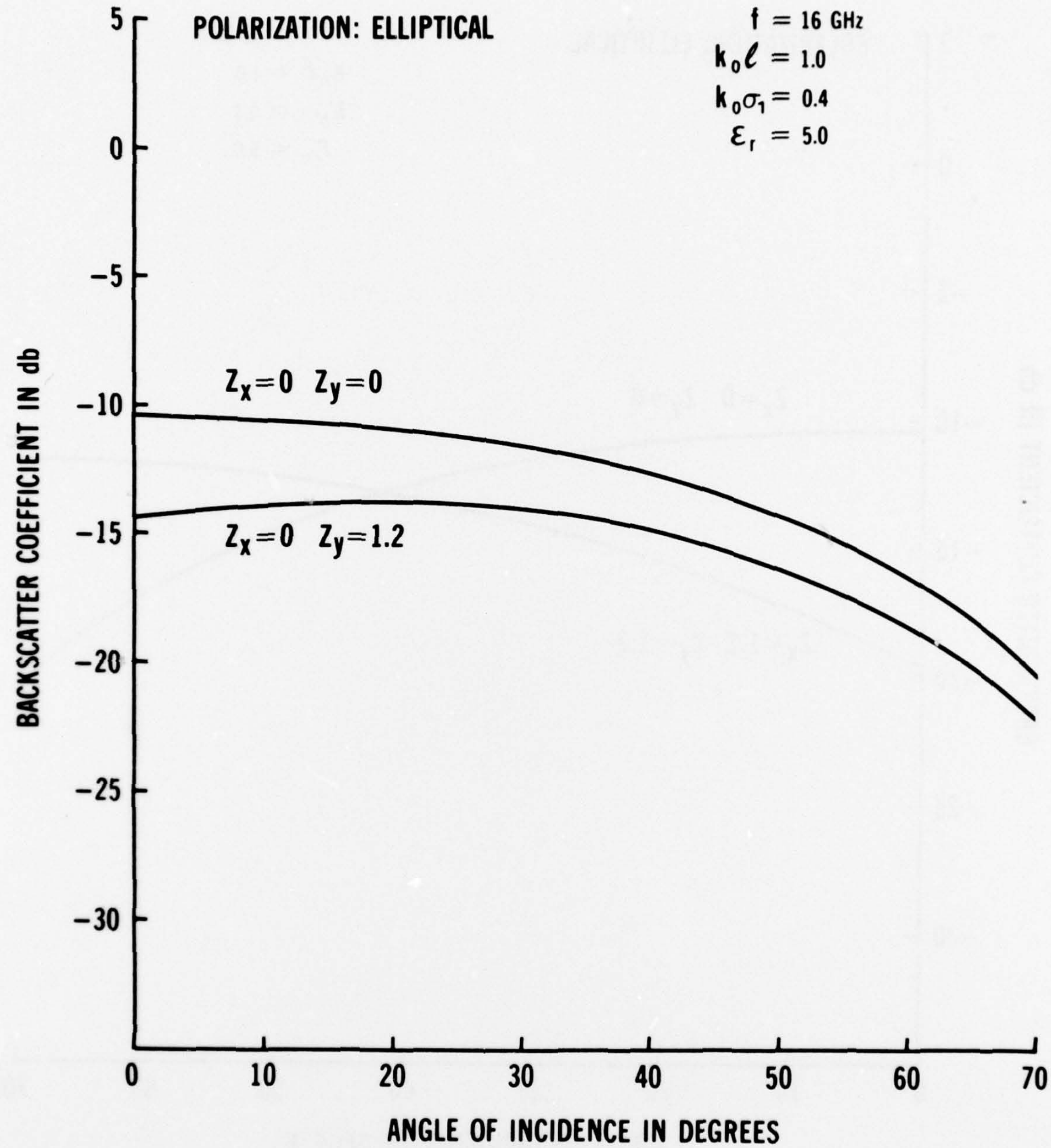

Figure 12. Study of Z_y Variations

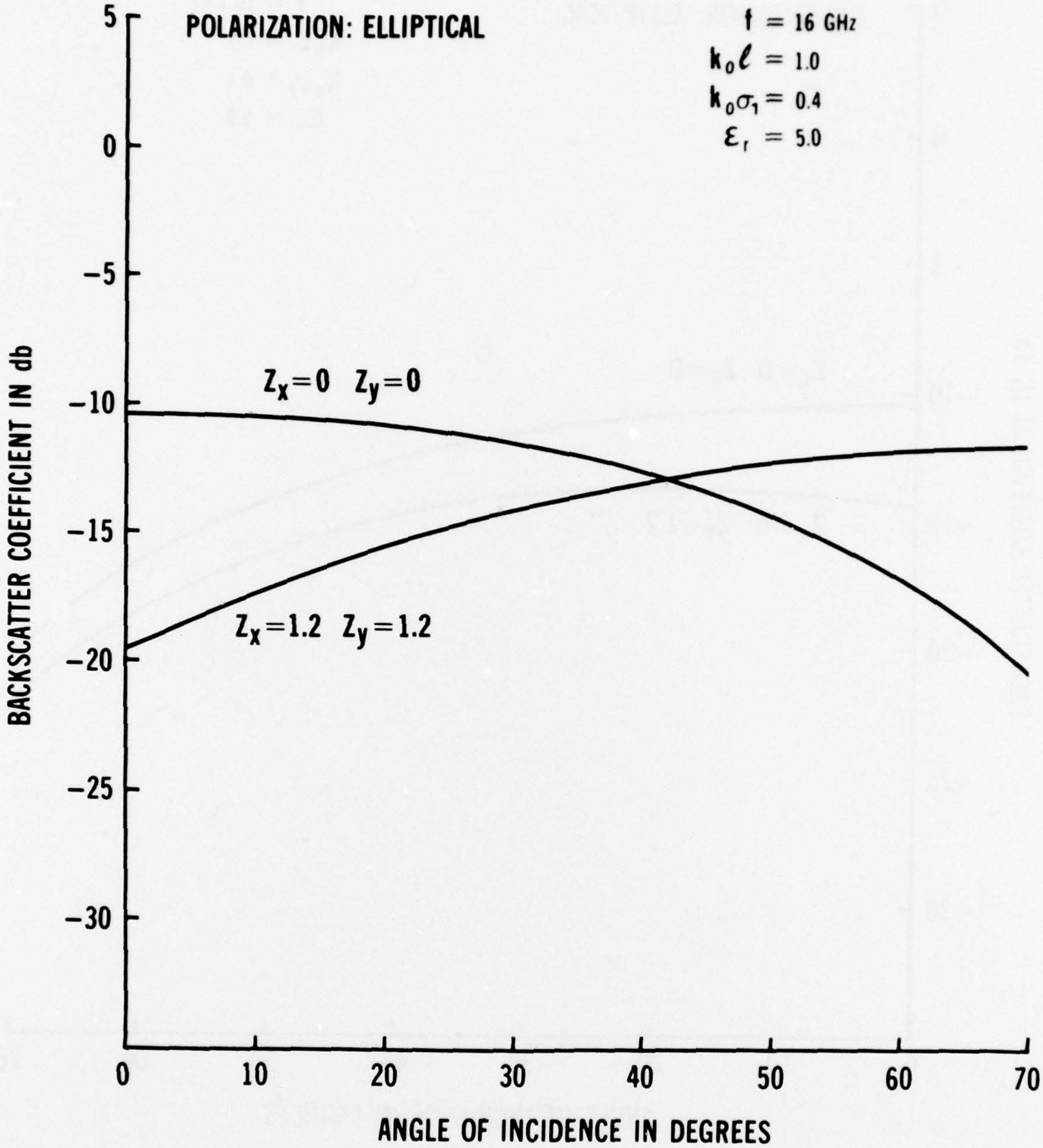

Figure 13. Study of Z_x and Z_y Variations

and $Z_x = 0$ is seen to be much lower than the curve associated with no slope, although it does have approximately the same shape. The effect of Z_y given in Figure 9 is similar to the effect exhibited in figure 6 for vertical polarization. In figure 10, the result of having a large slope in both the x and the y directions is seen. Now, we see that the crossover point is located at $\theta = 40°$.

In figures 11 through 13, the results for one type of elliptical polarization are seen. Figure 11 shows the influence of a large slope in x and no slope in y. Once again we have a crossover point at $\theta = 25°$. Figure 12 shows the effect of a slope in y and no slope in x. We can see that even though there is no crossover point the two curves do not have the same shape. Figure 13 presents the result of having a large slope in both x and y. In this figure, the crossover point now occurs at approximately $\theta = 42°$.

In this section, we have considered the influence of surface slopes on the calculations of the radar backscatter coefficient for four different polarizations. It was clearly evident that Z_x influences the calculation of $\sigma°$ in a different manner than Z_y. Also, we were able to see that the various effects of slope changed from one polarization to another. Since the influence of slope changes with polarization and also with the direction of the slope, any simple hueristic approach probably could not be developed to adequately explain the effect of slope on radar scattering in a correct manner.

CONCLUSIONS

The following conclusions can be made from the analysis and the curves presented in the last section.

1. A theory has been developed to explain radar backscattering from a slightly rough surface that has an arbitrary tilt. The effect of depolarization due to the tilted plane was considered, but depolarization due to the slightly rough surface itself was not considered.

2. The slope in the plane of incidence (Z_x) influences the calculation of the radar backscatter coefficient in an entirely different manner than a slope in a plane orthogonal to the plane of incidence (Z_y) for a given polarization.

3. The influence of a given slope of the tilted rough surface upon the calculation of the radar backscatter coefficient depends upon the polarization of the incident electromagnetic wave.

4. The results of this mathematical model derived for electromagnetic wave scattering from a tilted, slightly rough surface indicates that significant error could occur if only a single slope mathematical solution is applied to radar image simulation problems.

APPENDIX

A COMPUTER PROGRAM LISTING FOR THE CALCULATION OF THE RADAR BACKSCATTER COEFFICIENT

```
      PROGRAM SIGMAO(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C     THIS PROGRAM COMPUTES THE RADAR BACKSCATTER COEFFICIENT
C     FOR A SLIGHTLY ROUGH TILTED SURFACE
      COMPLEX QJ,QEIX,QEIZ,QA1,QA21,QA2,QC0,QC1,
     1QC2,QA11,QA22,QA33,QA44,QA,QAA11,QAA22,QAA33,
     2QAXH,QAYH,QAZH,QALP,QALPC,QSIGOA,QERX,QERZ
      READ(5,101)F,ZXP,ZYP,XKOL,XKOSIG,ER
  101 FORMAT(6F13.1)
      PI=3.1415926535898
      U=4.0E-7
      UO=PI*U
      EO=8.854E-12
      XK=2.*PI*F*SQRT(UO*EO)
      XL=XKOL/XK
      SIG1=XKOSIG/XK
      W=2.*PI*F
      THETA=0.0
      KK=8
      WRITE(6,201)F
  201 FORMAT(1H ,2X,2HF=,F15.2)
      WRITE(6,204)XKOL
  204 FORMAT(1H ,2X,3HKL=,F7.5)
      WRITE(6,205)XKOSIG
  205 FORMAT(1H ,2X,6HKSIG1=,F7.5)
      WRITE(6,206)ER
  206 FORMAT(1H ,2X,3HER=,F7.5)
      LL=4
      NN=4
      DO 12 I=1,LL
      DO 13 J=1,NN
      WRITE(6,202)ZXP
  202 FORMAT(1H ,2X,4HZXP=,F7.5)
      WRITE(6,203)ZYP
  203 FORMAT(1H ,2X,4HZYP=,F7.5)
      DO 14 N=1,KK
      TH2=THETA*PI/180.
      TANT=TAN(TH2)
      DDO=1./SQRT(1.+ZXP**2+ZYP**2)
      QJ=(0.0,1.0)
      QEIX=-COS(TH2)
      EIY=0.0
      QEIZ=-SIN(TH2)
      COSTP=(ZXP*SIN(TH2)+COS(TH2))*DDO
      SN=SIN(TH2)
      CN=COS(TH2)
      SINTP=SQRT(1.-COSTP**2)
      IF(ZYP)250,260,250
  250 D1=1./SQRT(ZYP**2+(SN-ZXP*CN)**2)
      QA1=D1*(ZYP*(QEIX*CN+QEIZ*SN)+EIY*(SN-ZXP*CN))
      QA21=(SN-ZXP*CN)*(QEIZ*SN+QEIX*CN)
      QA2=XK*(ZYP*EIY-QA21)*D1/(W*UO)
      ALX1=DDO*D1*(SN-ZXP*CN+SN*ZYP**2)
      ALX2=-DDO*D1*ZYP*(CN+ZXP*SN)
```

```
      ALX3=DDO*D1*(ZXP*(SN-ZXP*CN)-CN*ZYP**2)
      ALY1=ZYP*CN*D1
      ALY2=D1*(SN-ZXP*CN)
      ALY3=D1*ZYP*SN
      ALZ1=-ZXP*DDO
      ALZ2=-ZYP*DDO
      ALZ3=DDO
      GO TO 270
260   IF(ZXP)261,262,261
262   QA1=EIY
      QA2=-XK*(QEIZ*SN+QEIX*CN)/(W*UO)
      ALX1=1.0
      ALX2=0.0
      ALX3=0.0
      ALY1=0.0
      ALY2=1.0
      ALY3=0.0
      ALZ1=0.0
      ALZ2=0.0
      ALZ3=1.0
      GO TO 270
261   IF(ZXP-TANT)263,264,263
263   SNAB=(SN-ZXP*CN)/ABS(SN-ZXP*CN)
      QA1=EIY*SNAB
      QA2=-XK*(QEIZ*SN+QEIX*CN)*SNAB/(W*UO)
      ALX1=SNAB/SQRT(1.+ZXP**2)
      ALX2=0.0
      ALX3=ZXP*SNAB/SQRT(1.+ZXP**2)
      ALY1=0.0
      ALY2=SNAB
      ALY3=0.0
      ALZ1=-ZXP/SQRT(1.+ZXP**2)
      ALZ2=0.0
      ALZ3=1./SQRT(1.+ZXP**2)
      GO TO 270
264   QA1=EIY
      QA2=-XK*(QEIZ*SN+QEIX*CN)/(W*UO)
      ALX1=1./SQRT(1.+TANT**2)
      ALX2=0.0
      ALX3=TANT/SQRT(1.+TANT**2)
      ALY1=0.0
      ALY2=1.0
      ALY3=0.0
      ALZ1=-TANT/SQRT(1.+TANT**2)
      ALZ2=0.0
      ALZ3=1./SQRT(1.+TANT**2)
270   XKP=W*SQRT(UO*EO*EP)
      R1=COSTP*XKP**2-XK*SQRT(XKP**2-(XK*SINTP)**2)
      R2=COSTP*XKP**2+XK*SQRT(XKP**2-(XK*SINTP)**2)
      RPP=R1/R2
      TPP=1.+RPP
      QC0=QJ*(XK**2-XKP**2)*(1.-RPP)*COSTP/(2.*PI*XK)
      QC1=QJ*(2.*XK*SINTP*SINTP)*TPP*(XK**2-XKP**2)/(2.*PI*XKP**2)
```

```
      ETA=SQRT(U0/E0)
      XKZ=XK*COSTP
      XKZP=SQRT(XKP**2-(XK*SINTP)**2)
      QC2=-QJ*TPP*XK*SINTP*SINTP*(XK**2-XKP**2)/(2.*PI*XKP**2)
      QA11=-XKZ*((XK*XKZP)**2)*QC0
      QA22=-XKZ*XKZP*XK*XKP*XKP*QC1
      QA33=-XKZ*XKZP*XK*XKP*XKP*QC2
      QA44=XKZP*(XKZ*XKP)**2+XKZ*(XK*XKZP)**2
      QA=(QA11+QA22+QA33)/QA44
      QAA11=QA2*ETA*QA*COSTP
      RP1=XK*COSTP-SQRT(XKP**2-(XK*SINTP)**2)
      RP2=XK*COSTP+SQRT(XKP**2-(XK*SINTP)**2)
      RP=RP1/RP2
      TP=1.+RP
      QAA22=-QJ*QA1*TP*(XKP**2-XK**2)/(2.*PI*(XKZ+XKZP))
      QAA33=QA2*QA*ETA*SINTP
      QAXH=QAA11*ALX1+QAA22*ALY1+QAA33*ALZ1
      QAYH=QAA11*ALX2+QAA22*ALY2+QAA33*ALZ2
      QAZH=QAA11*ALX3+QAA22*ALY3+QAA33*ALZ3
      QERX=-CN
      ERY=0.0
      QERZ=-SN
      QALP=QAXH*QERX+QAYH*ERY+QAZH*QERZ
      QALPC=CONJG(QALP)
      WS=0.5*XL*XL*EXP(-(XK*XL*SINTP)**2)
      QSIGOA=8.*(PI*XK*SIG1*COSTP)**2*QALP*QALPC*WS
      SIGOA=REAL(QSIGOA)
      SIGOB=10.*ALOG10(SIGOA)
      WRITE(6,207)THETA,SIGOB
  207 FORMAT(/2X,6HTHETA=,F5.2,10X,17HSIGMA ZERO IN DB=,F7.2)
      THETA=THETA+10.
   14 CONTINUE
      THETA=0.0
      ZXP=ZXP+0.4
   13 CONTINUE
      THETA=0.0
      ZXP=0.0
      ZYP=ZYP+0.4
   12 CONTINUE
      STOP
      END
```

LIST OF SYMBOLS

j	is the square root of minus one ($\sqrt{-1}$).
f	is frequency.
ω	is $2\pi f$.
t	is time in seconds.
$\underline{n}_1$	is a unit vector in the direction of propagation.
θ	is the angle of incidence.
$\underline{a}_x, \underline{a}_y, \underline{a}_z$	are unit vectors in the x, y, and z directions, respectively.
θ'	is the local angle of incidence.
$Z(x, y)$	is the expression for the tilted plane.
$s(x, y)$	is the expression for the random slightly rough surface.
$\underline{n}$	is a unit vector which is normal to the surface $Z(x, y)$.
$\underline{n}_2$	is a unit vector in the direction of the receiver.
Z_x	is the slope of the tilted plane in the x direction.
Z_y	is the slope of the tilted plane in the y direction.
$\underline{E}_i$	is the incident electric field.
k_o	is the free space propagation constant.
$\hat{\underline{r}}$	is a position vector.
$\underline{a}'_x, \underline{a}'_y, \underline{a}'_z$	are unit vectors in the x′, y′, and z′ directions, respectively.
$\underline{e}_i$	is the unit polarization vector for the incident wave.
e_{ix}, e_{iy}, e_{iz}	are the components of $\underline{e}_i$ in the x, y, and z, directions respectively.
$\underline{H}_i$	is the incident magnetic field vector.
μ_o	is the permeability of free space.
k_x, k_y	are the Fourier transform variables.
$A_{x1}(k_x, k_y), A_{y1}(k_x, k_y), A_{z1}(k_x, k_y)$	are the amplitudes of the scattered waves in Fourier transform variables for a local horizontally polarized incident wave.
$R_\perp$	is the Fresnel reflection coefficient for a horizontally polarized incident wave.
k'	is the propagation constant in the medium below the surface.
ϵ_r	is the relative dielectric constant of the medium below the surface.
ϵ_o	is the permittivity of free space.
$T_\perp$	is the transmission coefficient for a horizontally polarized incident wave.

$A_{x2}(k_x, k_y)$, $A_{y2}(k_x, k_y)$, $A_{z2}(k_x, k_y)$ are the amplitudes of the scattered waves in Fourier transform variables for a local vertically polarized incident wave.

η is the intrinsic impedance of free space.

c is the speed of light in free space.

$R_{\parallel}$ is the Fresnel reflection coefficient for a vertically polarized incident wave.

$T_{\parallel}$ is the transmission coefficient for a vertically polarized incident wave.

E_s is the backscattered far field.

R is the distance from the origin of the local coordinate system (x', y', z') to the receiver in the far field.

$\underline{e}_r$ is the unit polarization vector for the received wave.

A_o is the area on the surface which is illuminated.

$\phi(\tau_x, \tau_y)$ is the correlation function of the random rough surface.

σ_1^2 is the variance of the undulations of the random rough surface.

$C(\tau_x, \tau_y)$ is the normalized correlation function of the random rough surface.

$W(k_x, k_y)$ is the surface roughness spectrum equal to the Fourier transform of $C(\tau_x, \tau_y)$.

ℓ is the correlation distance associated with $C(\tau_x, \tau_y)$.

σ° is the radar backscatter coefficient.

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